

Math 10B with Professor Stankova

Quiz 3; Tuesday, 2/6/2018

Section #203; Time: 9:30 AM

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Name: \_\_\_\_\_

Circle True or False or leave blank. (1 point for correct answer,  $-1$  for incorrect answer, 0 if left blank)

1. True **FALSE** The number of ways there are to flip 2 heads out of 5 total flips is  $P(5, 2)$  because the order of the coin flips matters.

**Solution:** We say order doesn't matter to mean that when picking the **location** of the  $H$ 's, the order in which we pick them does not matter and hence we should use  $C$ . So order always has to do with the order in which you choose things, not in the ordering compared to everything else.

2. **TRUE** False It is impossible to devise an algorithm to solve the stable marriage problem if men can marry other men (the "roommate problem").

**Solution:** Zvezda gave an example of a system of preferences such that there is no stable pairing which means that an algorithm cannot give you a stable pairing.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (4 points) How many ways can I buy 250 bubble teas from TeaOne for an event if there are 11 different options to choose from?

**Solution:** There are 250 indistinguishable balls which are the bubble teas, and the boxes are the options. So there are  $\binom{250+11-1}{250}$  ways to do this.

- (b) (4 points) The most popular option is original milk tea. How many ways can I do this if I need at least 20 of that option and at least 10 of every other option?

**Solution:** First I buy 20 of that option and 10 of all the other options. So I've bought 120 bobas and need to buy 130 more out of the 11 options. There are  $\binom{130+11-1}{130} \binom{140}{130}$  ways to do this.

- (c) (2 points) Suppose men and women have the preferences  $m_1 : w_1 > w_3 > w_2$ ,  $m_2 : w_3 > w_1 > w_2$ ,  $m_3 : w_3 > w_2 > w_1$  and  $w_1 : m_1 > m_2 > m_3$ ,  $w_2 : m_2 > m_3 > m_1$ ,  $w_3 : m_3 > m_2 > m_1$ . Is the matching  $(m_1, w_1), (m_2, w_3), (m_3, w_2)$  stable?

**Solution:** This is not stable because  $m_3$  prefers  $w_3$  to  $w_2$  and  $w_3$  prefers  $m_3$  to  $m_2$  which means that both will divorce their partners and get with each other.